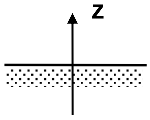
Solution exercice 3_1

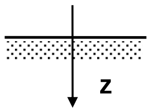
a) $\Psi = -h$ (Ψ : succion, h : charge de pression) ;
$\Psi_A = 780$ cm
$\Psi_B = 320$ cm

H : charge hydraulique, h : charge de pression
z : charge de gravité

b)



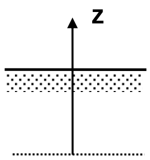
$H = h + z \Rightarrow$ $H_A = -780 + (-30) = -810$ cm
$H_B = -320 + (-50) = -370$ cm



$H = h - z \Rightarrow$ $H_A = -780 - 30 = -810$ cm
$H_B = -320 - 50 = -370$ cm

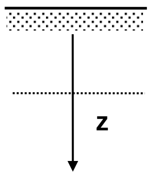
c) $\Delta H = H_A - H_B = -810 - (-370) = -440$ cm ou : $\Delta H = H_B – H_A = -370 - (-810) = 440$ cm
L'eau s'écoule dans le sens des charges décroissantes $\Rightarrow$ de B vers A (mouvement ascendant)

d)



$z_A = 0.5$ m
$z_B = 0.3$ m

$H_A = -780 + 50 = -730$ cm
$H_B = -320 + 30 = -290$ cm
$\Delta H = H_A - H_B = -730 - (-290) = - 440$ cm
$\Delta H = H_B – H_A = -370 - (-810) = 440$ cm



$z_A = -0.5$ m
$z_B = -0.3$ m

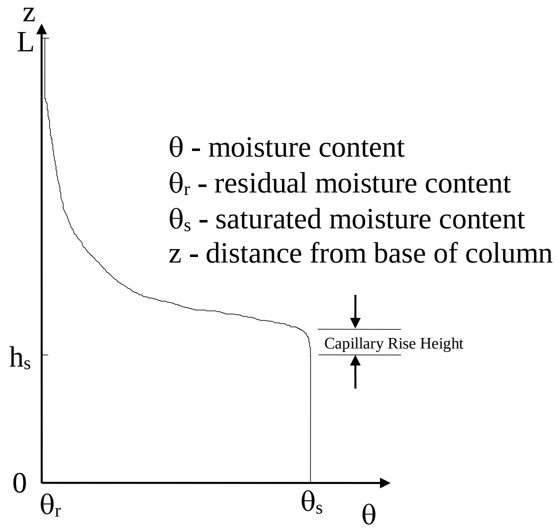
$H_A = -780 - (-50) = -730$ cm
$H_B = -320 - (-30) = -290$ cm
$\Delta H = H_A - H_B = -730 - (-290) = - 440$ cm
$\Delta H = H_B – H_A = -290 - (-730) = 440$ cm

On constate que, quel que soit le choix de l'orientation de l'axe des z, pour une référence altimétrique donnée, la charge hydraulique ne change pas. Par ailleurs, quel que soit le référentiel altimétrique choisi, la ***différence*** *de charge hydraulique reste* ***constante****.*

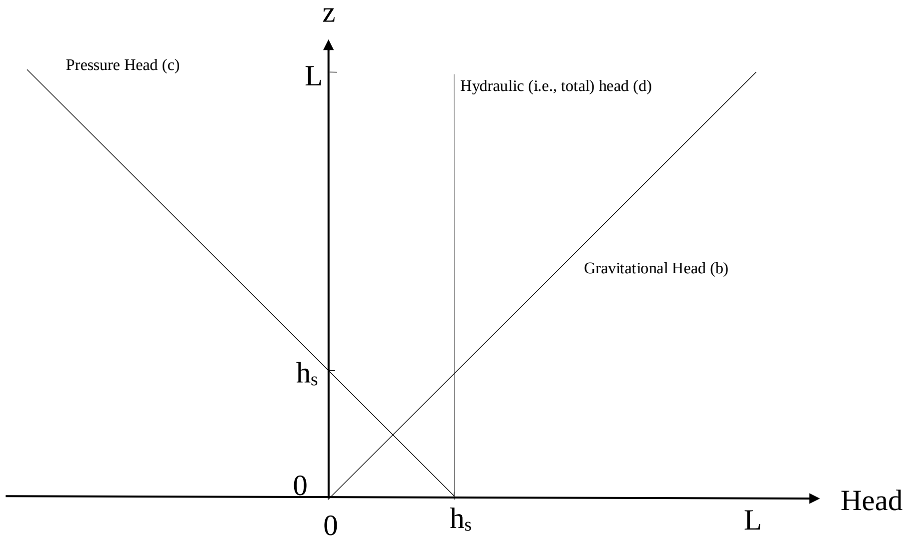
Solution exercise 3_2

Let the height of the saturated zone (to where the water pressure is atmospheric, as shown in the figure in the question) be h_s. Let $z = 0$ indicate the base of the column, and $z = L$ its top.

(a)

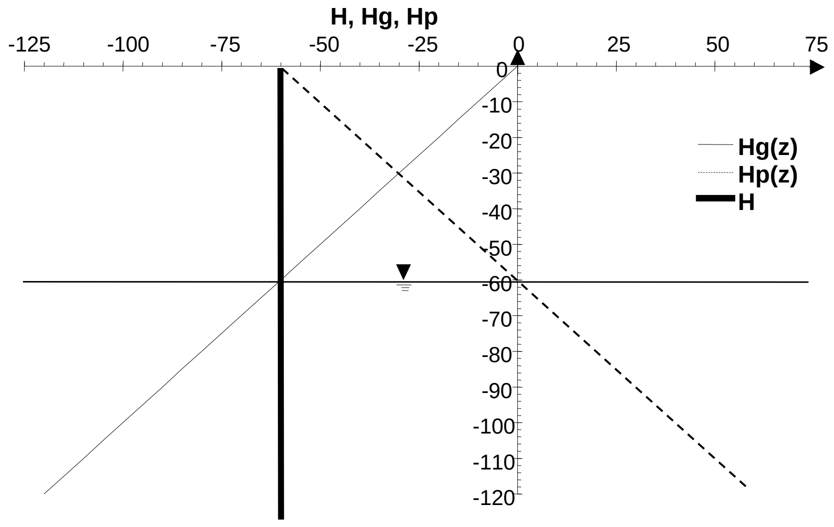


(b-d)



Solution exercice 3_3

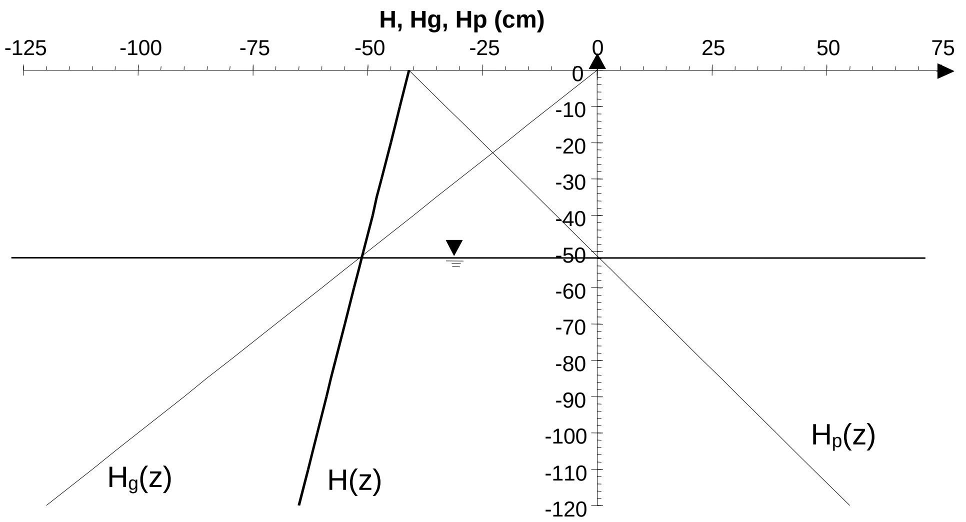
a) Cas hydrostatique :



H_g : charge de gravité (cm)
H_p : charge de pression (cm)
H : charge hydraulique (cm)

b) piézomètre 1 : $z_1 = -120$cm $h_1 = 55$ cm ➔ $H_1 = h_1 + z_1 = -65$cm
piézomètre 2 : $z_2 = -70$cm $h_2 = 15$cm ➔ $H_2 = -55$ cm

H n'est pas constante, donc il y a mouvement. Le flux est dans le sens du potentiel décroissant, donc dans le cas présent de 2 vers 1, soit vers le bas.



c) La nappe se trouve à une profondeur telle que la charge de pression h = 0, soit $H_p(z) = 0$.
Or, le profil de charge de pression $H_p(z)$ a pour équation: $z = a\, H_p(z) + b$
$-120 = 55\,a + b$ et : $-70 = 15\,a + b$ Il vient donc : $a = -1.25$ et $b = -51.25$
L'équation du profil de charge de pression $H_p(z)$ devient donc : $z = -1.25\, H_p(z) - 51.25$
Lorsque $H_p(z) = 0$, $z = -51.25$ cm. La nappe se trouve donc à env. 51cm de profondeur.

Solution exercice 3_4

	Succion ψ (bars)	Humidité θ (cm³/cm³)	
		Sol A	**Sol B**
Capacité de rétention θ_{cr}	0.33	0.125	0.32
Point flétrissement temporaire θ_{ft}	10	0.052	0.135

a) volumes nécessaires (détermination graphique):

Sol A: env. 3 mm soit env. 15 m^3
Sol B: env. 83 mm soit 415 m^3

Note: The solution is obtained by estimating the area between the existing moisture content and the desired moisture content. This area has dimensions $\Delta\theta \times$ depth. To compute the volume of water to be added, multiply by the surface area of the field (0.5 ha).

b)

RFU = $(\theta_{cr} - \theta_{ft}) \times 50$ cm

Condition initiale	Sol A	Sol B
θ_{ft}	env. 36.5 mm, soit env. 182 m^3	env. 92.5 mm, soit env. 462 m^3

Solution exercise 3_5

It is obvious that the scaling in these media relates to lengths. Thus, given length l_1 in medium #1 and l_2 in medium #2, these values are related by:

$$\frac{l_1}{\lambda_1} = \frac{l_2}{\lambda_2}.$$

(a) From the formula above we have:

$$d_2 = \frac{d_1 \lambda_2}{\lambda_1},$$

where the pore diameters are given by d_i, i = 1, 2.

(b) The definition of porosity is:

$$n = \frac{V_f}{V_t}.$$

Since the scaling is isotropic (does not change with direction), the relative volume of the fluid components (air and water) to total volume is unchanged by a magnification or contraction of the medium. Thus, the porosity is identical in each case.

(c) There will be relatively more smaller pores in medium #2. At a given value of h, these smaller pores will release proportionally less fluid than medium #1. Thus, the water content will be higher in medium #2 than in medium #1.

(d) In a capillary tube (radius r), the capillary pressure is just the height, z_h, of water in the tube. Thus, from the "équation de Jurin", the capillary pressure, p_c, is:

$$\frac{p_c}{\rho g} = z_h = \frac{2\sigma}{\rho g r}.$$

Recall that the capillary pressure if $p_c/\rho g = p_{air}/\rho g - p_w/\rho g = -p_w/\rho g$ since we take $p_{air}/\rho g = 0$. Also recall that the suction is given by $\psi = -p_w/\rho g$. Therefore, the above equation is:

$$\psi = -\frac{p_w}{\rho g} = -h = \frac{2\sigma}{\rho g r}.$$

A capillary with radius r_1 in medium #1 has the corresponding radius:

$$r_2 = \frac{r_1 \lambda_2}{\lambda_1}$$

in medium #2. Any given pore in medium #1, with radius r_1, will have a suction, ψ_1, given by:

$$\psi_1 = \frac{2\sigma}{\rho g r_1}.$$

The corresponding (scaled) pore in medium #2 will have a capillary pressure, ψ_2, of:

$$\psi_2 = \frac{2\sigma}{\rho g r_2} = \frac{2\sigma}{\rho g\left(\frac{r_1\lambda_2}{\lambda_1}\right)} = \frac{\lambda_1}{\lambda_2}\psi_1.$$

Now, we found in (b) that saturations in each medium is the same, when corresponding pores are filled. Put another way, if all pores up to a given size in medium #1 are filled with water (giving a known saturation), then all the corresponding pores in medium #2 are also filled, to give the same saturation in both cases. Therefore, the saturation in medium #1 when pores up radius r_1 are filled equals the saturation in medium #2 when pores up to $r_2 = r_1\lambda_2/\lambda_1$ are filled, i.e.,

$$S_1(\text{pores up to } r_1 \text{ filled}) = S_2(\text{pores up to } r_1\lambda_2/\lambda 1 \text{ filled}).$$

Since pore radius corresponds to capillary pressure head, h, by Jurin's equation, this last expression becomes:

$$S_1(\psi_1) = S_2\left(\frac{\lambda_1}{\lambda_2}\psi_1\right).$$

The soil moisture characteristic curve can be expressed, identically, in terms of pressure head of water (i.e., capillary pressure), or, as done here, suction, since the negative of suction is the pressure head and vice versa. So, the above equation can also be written in terms of pressure, i.e.,

$$S_1(h_1) = S_2\left(\frac{\lambda_1}{\lambda_2}h_1\right).$$